An Algebraic Framework for Updatable and Universal (zk)SNARKs

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4th ZKProof Workshop

Universitat Pompeu Fabra Barcelona

Obra Social Fundación "la Caixa"

Pairing-Based (zk)SNARKs

State of the art

Interactive Proof-Systems [GMR89]

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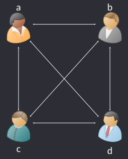
Multiparty Computation (Zcash Ceremony)

One ceremony per circuit !!!

Updatable and Universal SNARKs

- Multiparty Computation Model:



Updatable and Universal SNARKs

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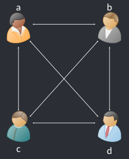

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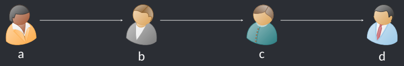

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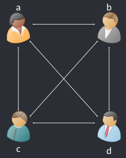

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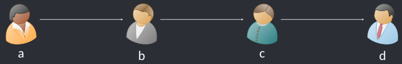

Circuit Specific vs. Universal

Motivation

Ishai's wisdom

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"This calls for a modular approach that allows for an easier navigation in the huge design space. A higher level of modularity and abstraction is useful (...) "

Updatable and Universal (zk)SNARKs

Common Design Principle

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Common Design Principle

AHP/
PHP/
...

Updatable and Universal (zk)SNARKs

Common Design Principle

$\boxed{\text{AHP/ PHP/ } \ldots} + \boxed{\text{Polynomial Commitment}}$

Updatable and Universal (zk)SNARKs

Common Design Principle

$$\boxed{\begin{array}{c}\text{AHP/}\\ \text{PHP/}\\ \ldots\end{array}} + \boxed{\begin{array}{c}\text{Polynomial}\\ \text{Commitment}\end{array}} = \boxed{\text{zkSNARK}}$$

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Holographic:

Updatable and Universal (zk)SNARKs

Common Design Principle

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Holographic:

- *Indexer* computes relation-dependent polynomials

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Sonic[MBKM19] — Plonk[GWC19] — Marlin[CHMMVW20] — Lunar[CFFQH20] — Claymore[SZ21]

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This proposal

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¿Can we break down further the information theoretic component?

Organization

- Break Circuit Satisfiability in three main algebraic components:

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  - Hadamard Product

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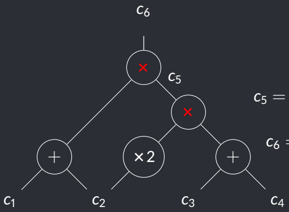
- Definition of Verifiable Subspace Sampling

Groth16: Overview Example

- Constraint system:



$$\begin{aligned} c_5 &= (2c_2)(c_3+c_4) \\ c_6 &= (c_1+c_2)c_5 \end{aligned} \iff (2c_2, c_1+c_2) \circ (c_3+c_4, c_5) = (c_5, c_6)$$

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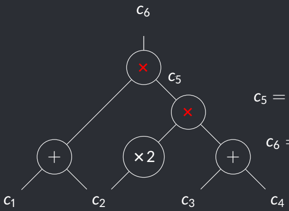

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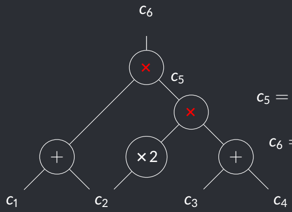

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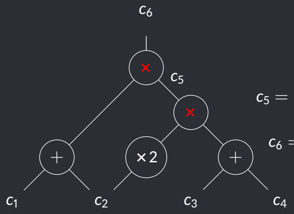

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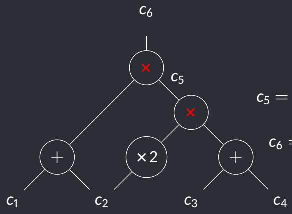

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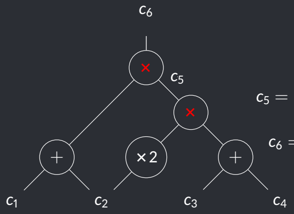

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Main Tool: “Compressed” Linear Algebra

Hadamard Product

[1] Completeness, soundness: $\mathcal{R}$ arbitrary set, efficiency: $\mathcal{R}$ multiplicative subgroup.

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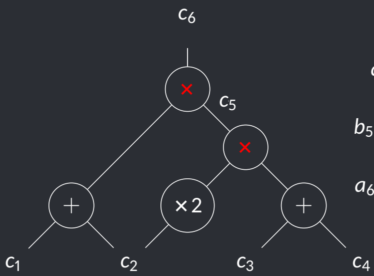

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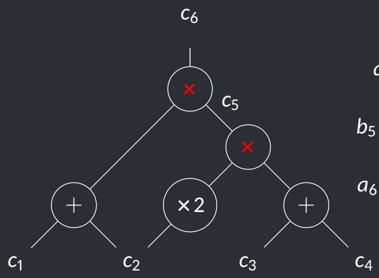

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a_i's left inputs, b_i's right inputs, c_i's outputs.

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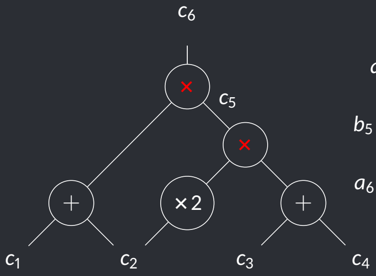

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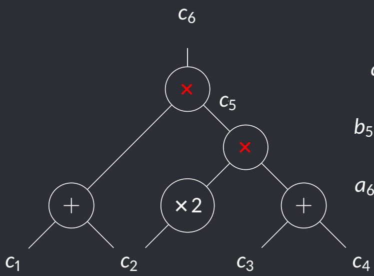

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Linear relations:

$$\begin{pmatrix} a_5 \\ a_6 \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix} \vec{c}, \quad \begin{pmatrix} b_5 \\ b_6 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \vec{c},$$

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Number of linear constraints: $2 \times \sharp$mult. gates

"Compressed" Linear Algebra

Inner Product

- Define $\mathbf{W} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & -\mathbf{F} \\ \mathbf{0} & \mathbf{I} & -\mathbf{G} \end{pmatrix}$, check that $\mathbf{W} \cdot (\vec{a}, \vec{b}, \vec{c}) = \vec{0}$

[2] Aurora's Univariate Sumcheck: Completeness, soundness, efficiency: $\mathscr{R}$ multiplicative subgroup. This work: New simple proof. Completeness, soundness: $\mathscr{R}$ arbitrary. Efficiency: $\mathscr{R}$ multiplicative.

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$\{\vec{w}_i(X)\}_{i=1}^{m}$ can be computed by the indexer, but 2m inner products

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Verifiable Subspace Sampling

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- Sample a random vector $\vec{d}$ in the rowspace of the matrix

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- Sample a random vector $\vec{d}$ in the rowspace of the matrix
- Check one inner product $\vec{d} \cdot (\vec{a}, \vec{b}, \vec{c}) = 0$.

Verifiable Subspace Sampling

Algebraic Intuition

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Verifiable Subspace Sampling

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Verifiable Subspace Sampling

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Verifiable Subspace Sampling

In the polynomial world

$$D(X) \quad = \left(\vec{\mathsf{Samp}}(x)^{\top}\mathbf{W}\right)\vec{\lambda}(X) = \vec{d} \cdot \vec{\lambda}(X)$$

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- *Verifier*: Who evaluates $P(X, Y) = \left(\vec{\mathsf{Samp}}(Y)^\top \mathbf{W}\right)\vec{\lambda}(X)$ in $Y = x$?

Verifiable Subspace Sampling

Definition

- *Offline phase:* Indexer outputs polynomials describing matrix $\mathbf{W}$.

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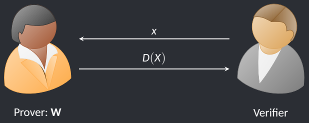

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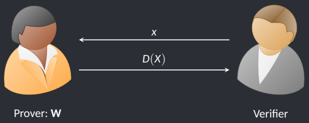

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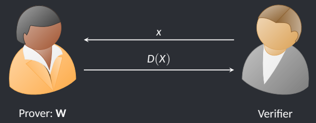

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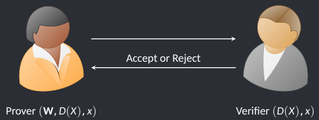

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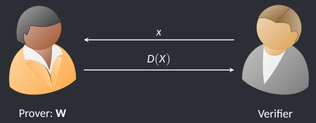

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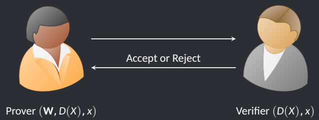

- *Decision phase:* Verifier accepts only if $D(X)$ encodes vector in the rowspace of $\mathbf{W}$ sampled according to x.

Verifiable Subspace Sampling

A common strategy

- Setup: Universal SRS

Verifiable Subspace Sampling

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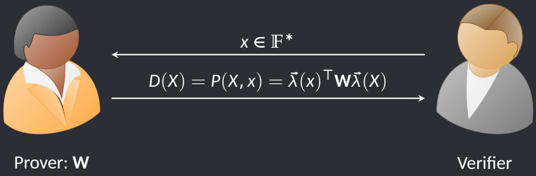

Verifiable Subspace Sampling

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Verifiable Subspace Sampling

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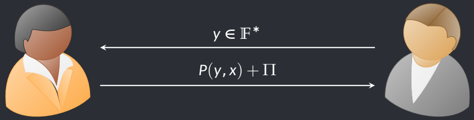

Prover $(\mathbf{W}, D(X), x)$

Verifier $(D(X), x)$, Output: 0/1

Verifiable Subspace Sampling

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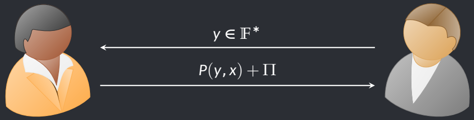

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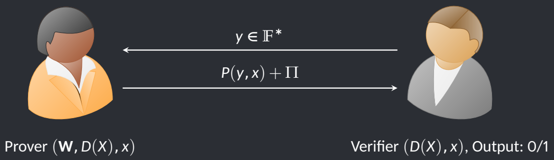

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- Π is a proof that $P(y, x)$ is correctly evaluated (Signature of Correct Computation of Sonic).

Verifiable Subspace Sampling

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Verifiable Subspace Sampling

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Verifiable Subspace Sampling

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Verifiable Subspace Sampling

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Thank you![3]

[3] I am looking for a research stay/internship :) arantxa.zapico@upf.edu